8. Granville, P. S., *J. Ship Res.*, 6, 43 (October, 1962).
9. Hunsaker, J. C., and B. G. Rightmire, "Engineering Applications of Fluid Mechanics," 1 ed., pp. 138-139, McGraw-Hill, New York (1947).
10. Kay, J. M., "An Introduction to Fluid Mechanics and Heat Transfer," 2 ed., pp. 71-72, 170-171, Cambridge Univ. Press, England (1963).
11. Knudsen, J. G., and D. L. Katz, "Fluid Dynamics and Heat Transfer," pp. 487-488, McGraw-Hill, New York (1958).
12. Metzner, A. B., *Ind. Eng. Chem.*, **49**, 1429 (1957).
13. Rohsenow, W. M., and H. Y. Choi, "Heat, Mass, and Momentum Transfer," pp. 42, 75, 197-200, 413-416, Prentice-Hall, Englewood Cliffs, N. J. (1961).
14. Schlichting, Hermann, "Boundary Layer Theory," 4 ed., Chap. VII, pp. 535-539, 539-541, 537, McGraw-Hill, New York (1960).
15. ———, in "Handbook of Fluid Dynamics," V. L. Streeter, ed., 1 ed., Section 9, pp. 37-41, McGraw-Hill, New York (1961).

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Characterization and Gain Identification of Time Varying Flow Processes

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A method for continuously estimating the gain of a flow process by sinusoidal perturbation is presented. The resulting output perturbation is correlated with a second sinusoid to generate periodically an estimate of the process gain. A method of implementing such an identifier on a small analog computer is described.

The experimental testing of this identifier computer with both a real process (a pH regulating system) and with an analog computer simulation of the process is described. The results of identification tests with a nonstationary system are presented. From these results it is concluded that the identifier estimates the process gain satisfactorily, introducing a delay (equal to one-half the period of identification) and making an effective sampling or clamping of the gain estimate (over each period of identification).

A large class of problems in the process field involves the control of complex, multivariable dynamic systems. From the point of view of control theory, the most desirable approach is to consider all the process variables; that is, the process outputs (state variables) are measured and the process inputs (manipulated variables) are regulated to control the system in some desired fashion. In practice this is seldom done; perhaps the major reasons for not using such an approach are that it is usually difficult to measure certain types of state variables (for example, chemical concentrations), and that the detailed knowledge of the process dynamics often is lacking.

More often, an easily measured output (such as temperature) and an easily manipulated input (a stream flow rate) are chosen. The approximate process dynamics relating these two variables are obtained (often this step involves experimental testing) and a conventional control system is installed. Considering the system as a single input/single output process* has many obvious advantages, notably the relative ease of control system design. The disadvantage of this approach is that the effects on the process dynamics of the disregarded process variables may be important. Hence a controller designed for one set of process conditions may not be satisfactory if the conditions change.

An alternative approach which may be used with flow systems is to retain the single input-single output formulation of the process, but to take into account the possibility of time variation in process characteristics which may result from variation in the unmeasured state variables. For example, a stirred tank chemical reactor may be controlled by measuring the temperature of the reacting mixture and by adjusting cooling water flow rate to maintain this quantity fixed. However, the process dynamics are known to be functions of the contents of the reactor and hence will vary with the concentrations of reactant, product, and catalyst.

The problem, then, becomes one of measuring process dynamics continuously and adjusting the controller parameters to compensate for variation in process dynamics. Such a control approach is generally termed adaptive; the controller adapts itself to maintain satisfactory control in spite of a time varying process.

This paper treats a subclass of such systems: gain varying flow processes. Thus, approximate mathematical analysis of a chemical reactor, such as that mentioned above, can be effected by considering the system equations linearized about the operating point. This may indicate the possibility of time variation in the process gain. The mathematical analysis in the next section will show the *p*H regulating system to be such a process.

A particular method of determining the process gain will be used; that is, the control system input will be perturbed sinusoidally. The resulting (approximately) sinusoidal perturbation in the measured system output will furnish a periodic estimate of the process gain. The necessary theory will be developed and experimental verification furnished. A companion paper will consider the problem of constructing a control system to use such information.

* Single input in the sense that there is only one manipulated input.

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In an aqueous system consisting of a number of buffer species, the relationship between pH and the amount of strong base (or acid) added may be expressed by

$$pH = pH_1\,(x_B, X) \tag{1}$$

Because the ionization constants relating the concentrations of certain groups of these species provide constraints which must be satisfied in the system, pH may be related to all independent concentration variables by an expression of the form

$$pH = pH\,(c_B, X') \tag{2}$$

where X' consists of the concentration of all independent buffer species. Thus the phosphoric acid system (H_3PO_4, $H_2PO_4^-$, $HPO_4^=$, $PO_4^{\equiv}$) has only one independent buffer concentration variable, because there are three ionization constants relating the concentrations of the four buffer species. Hence, for this single acid system, $X' = x'$ where x' will be the total concentration of all phosphoric species (g.-moles/liter).

The reason for introducing the functional relation between pH and c_B may be seen from Figure 1. Here titration curves of two aqueous systems with different buffer levels X' have been drawn. In the typical pH feedback control system, pH is the measured output variable and c_B is related to the control input or manipulated variable. The parametric vector X' represents a group of important, but unmeasured, system state variables.

Consider the representative pH control system shown schematically in Figure 2. Two streams enter the stirred tank, the controlled stream at volumetric flow rate G, and the controlling or manipulated stream at flow rate g. To derive the differential equations describing the process, consider that a strong base is being used as the control reagent to maintain the output of the tank at a fixed pH value, pH_s. The controlled stream, which contains the buffer species X, will enter at a pH less than or equal to pH_s. It is desired to know at any time the concentration of control reagent in the tank C_B. To do this it will be assumed that the concentration of base in the controlled or load stream C_{BL} may vary. This deviation in the controlled stream constitutes a load variable which must be compensated for by changing g, the flow rate of the manipulated stream which enters at concentration C_{BM}.

If perfect mixing and constant volume are assumed, a material balance may be made

$$GC_{BL} + gC_{BM} - (G+g)\,C_B = V\frac{dC_B}{dt} \tag{3}$$

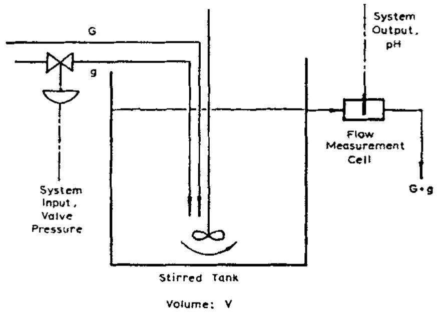

Fig. 2. Schematic representation of a pH control process.

For convenience, the C_B variables will be normalized with respect to C_{BM} and these normalized concentrations will be denoted by lower case letters so that c_{BM} is then equal to one and c_B and c_{BL} will have the units: volume control reagent/volume solution.* This allows the use of the standard titration curve (as ordinarily defined) in relating pH to c_B. Equation (3) then takes the form

$$V\frac{dc_B}{dt} = g - (G+g)\,c_B + Gc_{BL} \tag{4}$$

If the following deviation variables are introduced into Equation (4)

$$c_{BL}^* = c_{BL} - \overline{c}_{BL} \tag{5}$$

$$g^* = g - \overline{g}$$

$$c_B^* = c_B - \overline{c}_B$$

and higher order terms neglected, the result is a linearized form of Equation (4):

$$V\frac{dc_B^*}{dt} = -\,(G+\overline{g})\,c_B^* + (1-\overline{c}_B)\,g^* + G\,c_{BL}^* \tag{6}$$

For a given X' the relationship between pH and c_B might be as shown in Figure 3. In the vicinity of $\overline{c}_B$

$$pH - \overline{pH} = \left.\frac{d(pH)}{dc_B}\right|_{\overline{c}_B} (c_B - \overline{c}_B) \tag{7}$$

* For example, if the control reagent is 6 N KOH solution, the units of c_B are ml. of 6 N KOH solution per liter of total solution.

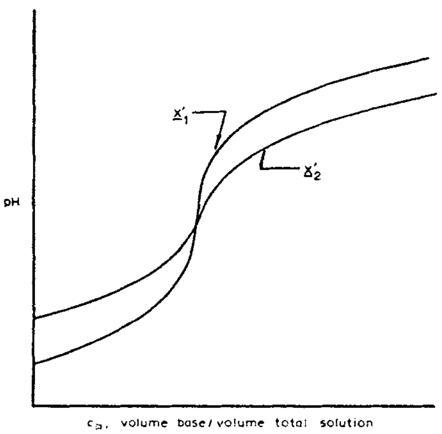

Fig. 1. Typical pH, c_B curves.

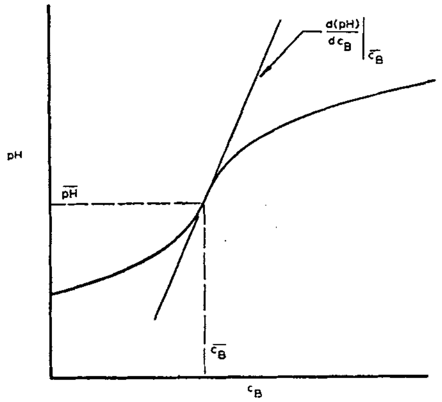

Fig. 3. pH vs. c_B for fixed X'.

Defining a deviation variable for pH

$$pH^* = pH - \overline{pH} \tag{8}$$

Equation (7) may be written

$$pH^* = Sc_B^* \tag{9}$$

where

$$S = \left. \frac{d(pH)}{dc_B} \right|_{\bar{c}_B} \tag{10}$$

Equation (6) now may be rewritten

$$\frac{d(pH^*)}{dt} + \frac{(G+\bar{g})}{V} pH^* = \frac{S(1-\bar{c}_B)}{V} g^* + \frac{SGc_{BL}^*}{V} \tag{11}$$

With the use of the Laplace transform of Equation (11) one obtains the result

$$pH^*(s) = \frac{S}{Ts+1} [K_p g^*(s) + K_L c_{BL}^*(s)] \tag{12}$$

where

$$T = V/(G+\bar{g})$$

$$K_p = (1-\bar{c}_B)/(G+\bar{g})$$

$$K_L = G/(G+\bar{g})$$

Equation (12) should not be regarded as an ordinary transfer function, because it contains the time-varying parameter S, which reflects the buffer level X′. This transfer function will be approximately valid for slow changes in S.

The dynamic characterization of the pH measurement may be accomplished as follows. The liquid leaving the tank will be assumed to pass in plug flow through a length of tubing into the measurement cell. Then

$$pH_o^*(s) = pH^*(s)\, e^{-T_D s} \tag{13}$$

Previous work by Marcikic (*1*) indicates that the pH cell may be characterized over small deviations in pH by a first-order transfer function

$$pH_m^*(s) = \frac{pH_o^*(s)}{T_m s + 1} \tag{14}$$

A block diagram depicting the linearized time-invariant version of the process may now be drawn as in Figure 4. The controller transfer function is given as $K_c G_c(s)$ where K_c is the steady state controller gain and $G_c(s)$ is any desired function of the Laplace operator s.

The control system is designed to minimize pH variation caused by load changes (variation in c_{BL}). The closed-loop transfer function relating pH to load change for a constant set point $\overline{pH}(s) = 0$, which may be obtained from the block diagram by standard methods, is

$$\frac{pH^*(s)}{c_{BL}^*(s)} = \frac{\dfrac{K_L S}{Ts+1}}{1 + \dfrac{G_c(s)\, K_p\, K_c S\, e^{-T_D s}}{(Ts+1)(T_m s+1)}} \tag{17}$$

From this result the approximate effect that changes in the gain S will have on control properties may be estimated. If "reasonably" slow changes in S are present (X′ changes slowly), then Equation (17) serves as a reasonable approximation to the nonlinear, nonautonomous case. Both the steady state gain and the characteristic function are seen to be modified by a variation in S. If large ultimate changes in S are possible, the dynamic characteristics of the system will obviously change, with the result that the system response will become unstable or too sluggish, depending on whether S increases or decreases. This presents two problems for the adaptive control system: identification of the change in S, and modification of the controller to compensate for the identified change in S. Only the first problem will be considered in the present paper; the second problem is the subject of a companion paper.

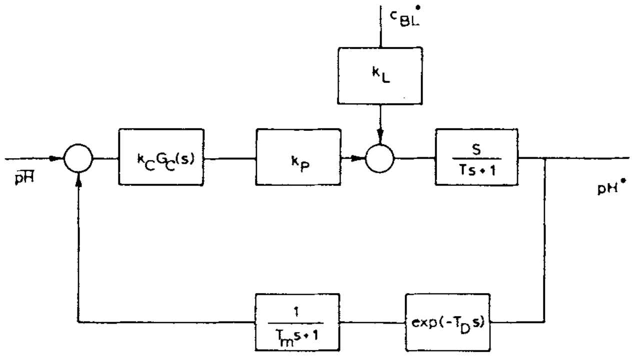

Fig. 4. Block diagram of linearized, time-invariant pH control system.

IDENTIFICATION OF THE GAIN OF A DOMINANTLY FIRST-ORDER PROCESS

Much literature on process characterization or identification has been published. A large portion of this work has been concerned with electrical and aerospace problems and the approach has been predominantly one wherein the effects of unknown loads on the system can be neglected. Most problems in the process field do not fall into this category.

Several papers concerned with identification of chemical processes have appeared. Angus and Lapidus (*2*) have given the results of identification of both the gain and dominant time constant of a chemical flow system. They employed a medium-sized digital computer to analyze the operating records of the process. A perturbing input to the process was used and this was correlated with output to estimate the process transfer function. The results indicated that a considerable length of operating time is required to estimate the process characteristics accurately; during this time the system must be stationary. Thus forty time constants of system operating time were required to obtain the gain and time constant within 10% of the correct values. For on-line identification of a nonstationary process, such as the pH system discussed previously, the assumption of stationarity over such a length of time is unreasonable.

Box and Chanmugan (*3*) proposed a different technique to estimate process gain. Essentially, they proposed the use of a small sinusoidal perturbation of the process input; the resulting process output perturbation was correlated with a sinusoid of the same frequency and phase to obtain a function which is linearly related to the process gain.

A modification of this technique was used in this work. The following advantages are obtained. The method is readily adapted to an analog computer, although a considerable amount of special logic circuits must be built. The time of identification is reasonable, that is, on the order of the system basic time constant.

However, a disadvantage is that the effect of unknown load changes on the system must be eliminated since these cannot be included in the analysis. With some flow systems process dynamics are functionally related to the material in the system at any time (as has been shown to be the case with a pH system). In this case it is possible to eliminate the effects of load changes by separating physically the control and identification functions, that is, by controlling and identifying in two separate tank systems. This topic will be discussed in the companion paper. For the present we shall assume that a load variable (of the type designated c_{BL}^* in the pH system) is not present in a general first-order system for which it is desired to identify the system gain. A delay will also be included in this analysis to increase its generality.

Consider a process of the following nature:

$$T \dot{z} + z = x\,(t - T_d)$$

and

$$y = kz$$

so that

$$T \dot{y} + y = kx\,(t - T_d) \tag{18}$$

Such a process can be related to the pH system studied previously by noting that k is the process gain which corresponds to S and may vary with time.

If this system is perturbed sinusoidally about some steady state point $(\bar{x}, \bar{y})$, beginning at $t = 0$

$$x^*(t) = A \sin \omega t \tag{19}$$

where $x^*(t)$ is the deviation of x about $\bar{x}$, then the system output deviation, after sufficient time (say $t > 5T$) for transients to become negligible, if k is assumed to remain constant, is

$$y^*(t) = \frac{Ak}{1 + \omega^2 T^2} \left[-\,\omega T \cos \omega(t - T_d) + \sin \omega(t - T_d)\right] \tag{20}$$

Equation (20) will be referred to as the ultimate periodic solution.

It is seen that the amplitude of $y^*(t)$ is proportional to k, the process gain. If it can be assumed that k varies slowly, the amplitude of $y^*(t)$ could be used as a measure of $k(t)$. The amplitude of a sinusoid is somewhat difficult to measure automatically. More easily accomplished with the aid of an analog computer is the evaluation of a definite integral of $y^*(t)$, which would give a quantity related to k.

If we consider an integral of the form

$$Z' = \int (B \cos \omega t + C \sin \omega t)\,(y^*(t) + \bar{y})\,dt \tag{21}$$

evaluated over one complete cycle, it can be seen from Equation (20) that $Z' = Z'(T, T_d, A, B, C, \omega, k)$. For fixed values of T, T_d, A, B, C, and ω, the quantity Z' is a linear function of k alone and can be used to estimate k.

Because a definite integral is evaluated, information concerning k is generated periodically with period $T_k = 2\pi/\omega$. To obtain information about k frequently, the period T_k should be small, which means that the chosen ω should be large; however, to minimize signal attenuation in the process, ω should be small. A compromise must be reached. For all experimental studies connected with this work, the value chosen was $\omega = 4\pi/T$ which means that $T_k = T/2$. In other words, the period of the sinusoidal perturbation is half the basic time constant of the process to be identified and the resulting attenuation is approximately 1/13.

For the specific experimental system used in this study (basically first-order with a small delay), this choice of ω gives rise to a phase shift in output signal of approximately $\pi/2$ radian with respect to input. (It should be noted that a phase shift of $\pi/2$ radian is not required by the method but is an experimental convenience.) To maximize signal retention in evaluation of Z', the correlating signal $(B \cos \omega t + C \sin \omega t)$ should be exactly in phase with $y^*(t)$. Resolved components of $y^*(t)$ which are 90 deg. out-of-phase with the correlating signal and, in addition, the average value $\bar{y}$, make no contribution to Z'. Because the phase shift which occurs in the experimental process is approximately $\pi/2$ with the value of ω indicated above, a modified integral was used

$$Z = \int_{t_{n_1} = \frac{2n\pi}{\omega} - \frac{\pi}{2\omega}}^{t_{n_2} = \frac{2n\pi}{\omega} + \frac{3\pi}{2\omega}} (B \cos \omega t)\,(y^* + \bar{y})\,dt \quad n = 0, 1, 2, \ldots \tag{22}$$

The reasons for choosing the limits shown in Equation (22) will be discussed later in this section.

Substitution of Equation (20) into this integral gives

$$Z = -\frac{ABk\pi}{\omega(1 + \omega^2 T^2)}\,(\omega T \cos \omega T_d + \sin \omega T_d) \tag{23}$$

A further simplification can be made with the system used in this study where $T_d << T$ and $\omega T_d \to 0$.

$$Z = -\frac{ABk\pi T}{1 + \omega^2 T^2} = -\frac{DkBT_k}{2} \tag{24}$$

where $D = \dfrac{A\omega T}{1 + \omega^2 T^2}$ is the approximate amplitude of $z^*(t)$ and Dk is the approximate amplitude of $y^*(t)$.

Thus it has been shown that the quantity Z varies linearly with k if all other parameters are constant. Such a situation is not entirely satisfactory in that the amplitude of $y^*(t)$ varies with k for constant amplitude of $z^*(t)$. This does not always lead to a good estimate of the slope of a nonlinear curve (particularly those of the form seen in pH systems). Figure 5a indicates two situations which might occur. At point 1 $(\bar{z}_1, \bar{y}_1)$ a good estimate is obtained; this is not the case at point 2. If the amplitude D, defined in Equation (24), is decreased to a value which gives a good estimate in the high slope region, the output amplitude would be far too small to measure Z accurately in the low slope region, with noise present in the measured variable.

To alleviate this difficulty, the output deviation amplitude Dk was maintained constant by changing the input deviation amplitude A to counteract changes in k. This situation is illustrated in Figure 5b. In both these figures, operation at two different values of y is indicated for the sake of clarity. Actually, y will always be fixed at the set-point of the controlled system. It was demonstrated in reference 4 that the amplitude of the input sinusoid $x^*(t)$ can be changed from A to a new value A' at the instant when the output equals its average value, with the result that the output will continue to be sinusoidal, uncontaminated by any transients. This property of the system is used to maintain the output amplitude Dk approximately constant in spite of slow variation in k.

It is now possible to specify an identification algorithm for a process with slowly varying gain of the nature discussed. Assume that a value of output deviation amplitude can be found which gives satisfactory estimation of the slope of the process curve in question. Such a value is indicated in Figure 5b as D_1k_1 or D_2k_2. Corresponding to this choice of output amplitude is a distinct value of Z, say $\tilde{Z}$, defined by Equation (24). Now, from Equation (24) it is seen that there are any number of combinations

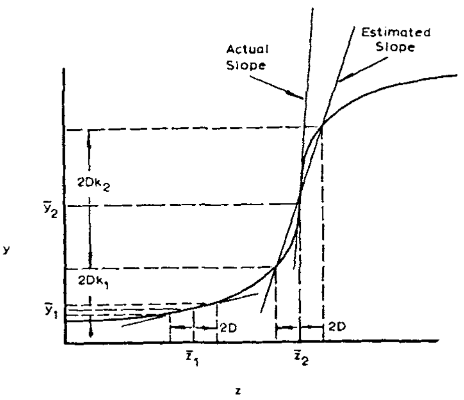

Fig. 5a. Slope estimation with constant amplitude of $z^*(t)$.

of A, k which result in $\tilde{Z}$. Let one of these be designated $\tilde{A}$, $\tilde{k}$. The equation for A may then be written

$$A = \tilde{A}\,\tilde{k}/k \tag{25}$$

where the product $\tilde{A}\,\tilde{k}$ is a constant.

To estimate k, the following relation is employed:

$$\hat{k}_n = (Z_{n-1}/\tilde{Z})\,\hat{k}_{n-1} \tag{26}$$

Equation (25) can be rewritten in discrete form

$$A_n = \tilde{A}\,\tilde{k}/\hat{k}_n \tag{27}$$

The operational sequence is as follows. At the end of the integration period T_k, Z is obtained. This is designated Z_{n-1}. Equation (26) is a correction used to furnish a new estimate of k based on Z_{n-1} and the estimate of k employed in the previous interval. Equation (27) furnishes the new value of the input perturbing amplitude. All these operations are carried out at the switching instant, which is designed to be at the time the system output signal is at its average value. This guarantees that no transient will be generated in system output. The choice of integration limits used in Equation (22) also is consistent with this requirement, that is, the integral is evaluated over a full cycle between average value crossings of the output deviation.

This identification method can be handled readily by a small analog computer and has been developed with this intention. Before proceeding to a discussion of the experimental realization of this identification method, it is possible to predict its dynamic characteristics from a brief analytical study.

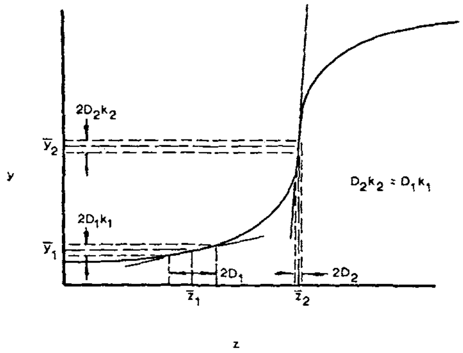

Fig. 5b. Slope estimation with constant amplitude of $y^*(t)$.

For stationary system operation it can be seen from the particular form of Equation (26) that the identifier will continue to estimate the system gain until the correct value is obtained. One or two identification periods are required to do this. For a specific assumed time variation in k, it is possible to evaluate the identifier response by means of Equations (22), (26), and (27).

Consider a linear change in system gain with time*

$$k = Pt + Q$$

or

$$k = Q\left(\frac{pt}{T_k} + 1\right)$$

where $p = PT_k/Q$ is some dimensionless constant. Evaluation of Equation (22) yields

$$Z[(n + 0.75)T_k] = \frac{-A_n B T \pi Q}{1 + \omega^2 T^2}[1 + 0.281p] \tag{28a}$$

$$n = 0$$

$$Z[(n + 0.75)T_k] = \frac{-A_n B T \pi Q}{1 + \omega^2 T^2}[1 + (n + 0.25)p] \tag{28b}$$

$$n = 1, 2, 3, \ldots$$

A_n is evaluated by Equation (27).

These equations may be solved for some value of p (say $p = 0.5$). The results for this case are given in Table 1.

Note that the value of $\hat{k}_n$ used during each interval is obtained just at the end of the previous interval. Therefore, to estimate the identification delay $k(t_{n2})$ must be compared with $\hat{k}(t_{n2})$, that is, the process gain is $1.375Q$ just at the *instant the identifier estimates it to be* $1.141Q$, etc. The difference is seen to be a constant, $0.25Q$, after the first complete period of unstationary operation. Hence $\hat{k}$ can be considered to be k delayed $T_k/2$ time units and sampled every T_k time units. Similarly, it can be shown that any linear variation in process gain eventually results *in this same delay between the actual process gain and* the identifier estimate of process gain (obtained just at the end of the identification cycle).

The presence of the identification delay $T_k/2$ plus the holding effect of the identifier over a complete identification cycle results in estimates of process gain which are as much as $3T_k/2$ time units behind the process with a linear gain variation. Whether or not this delay can be tolerated in a control loop is discussed in a companion paper.

EXPERIMENTAL TESTING OF THE IDENTIFICATION ALGORITHM

To test the identification theory as presented, both an experimental flow system of gain varying nature and a fairly sophisticated computing device are needed. A stirred tank pH regulating system corresponding to the example discussed earlier was constructed. The identifier was implemented by use of a small analog computer. The general features of both these systems will be discussed here; details are available in reference *4*.

Identification Flow System

Figure 6 presents a view of the experimental equipment. Essentially, a stirred tank (blending stage) was used in this

* Although a linear (ramp) change in an ordinary process *input* variable is not a severe test of such a control system (compared with a step or pulse input), a linear change in process *gain* can approximate the most drastic variation such a continuous system would encounter. Note that no limit has been placed on the slope of the ramp.

Table 1. Characterization of the Identifier

Linear gain change ($p = 0.5$)

n	Period of unstationary operation	Process gain, $k(t_{n2})$	A_n	$Z(t_{n2})$	Z_{n-1}	$\hat{k}_{n-1}$	$\hat{k}_n$	$\hat{k}(t_{n2})$
0	First	$1.375Q$	A_o	$1.141\tilde{Z}$	$\tilde{Z}$	Q	Q	$1.141Q$
1	Second	$1.875Q$	$0.876A_o$	$1.423\tilde{Z}$	$1.141\tilde{Z}$	Q	$1.141Q$	$1.625Q$
2	Third	$2.375Q$	$0.615A_o$	$1.308\tilde{Z}$	$1.423\tilde{Z}$	$1.141Q$	$1.625Q$	$2.125Q$
3	Fourth	$2.875Q$	$0.471A_o$	$1.235\tilde{Z}$	$1.308\tilde{Z}$	$1.625Q$	$2.125Q$	$2.625Q$
4	Fifth	$3.375Q$	$0.381A_o$	$1.191\tilde{Z}$	$1.235\tilde{Z}$	$2.125Q$	$2.625Q$	$3.125Q$
5	Sixth	$3.875Q$	$0.320A_o$	$1.160\tilde{Z}$	$1.191\tilde{Z}$	$2.625Q$	$3.125Q$	$3.625Q$

$$t_{n2} = (n + 0.75)T_k$$

The argument t_{n2} is used to indicate the value of the variable obtained just at the end of the identification period.

process to mix two incoming streams. One of the streams constituted the load and entered the tank with a certain pH (corresponding to some value of control reagent c_{BL}) and with buffer species concentration vector $\mathbf{x}'$. Provision for feeding this stream at constant flow rate was incorporated in the design. A second stream, the control reagent, entered the stirred tank through an automatic valve which controlled the flow rate g of this stream.

A number of tests were performed with this system to determine the characteristics of the identification method when operated in conjunction with a real process. Both stationary and nonstationary operations of the pH system were used; in the latter case a step change in $\mathbf{x}'$ was introduced in the load stream by switching from one feed solution to another. Facilities for mixing and storing the corrosive solutions required for these tests were provided.

The necessary input-output transducers were provided to enable use of the analog computer in conjunction with the stirred tank system. The system output, pH, was converted to voltage of such magnitude that the signal could be manipulated in the analog computer. In addition, a voltage-pressure transducer was used to convert computer volts to pressure which was applied to the automatic valve as the system input.

Identification Computer

Design. A large amount of logic must be built into an analog computer to use it to perform process gain identification with the algorithm given previously. Considerable effort was put into the design of satisfactory circuits for this purpose. (These circuits will be hereafter referred to as the "identification computer," even though only a portion of the equipment in one analog computer console was actually required to build them. The remaining equipment was available for simultaneous simulation or other purposes.)

Figure 7 is a block diagram of the identification computer in its simplest form. An important part of this circuit is the sine-cosine generator. This furnishes the perturbing signal to the process ($A \sin \omega t$), the correlating signal ($B \cos \omega t$) used to multiply the process output deviation, and a timing signal to the logic circuits ($B \cos \omega t$). Other than the logic circuits, the rest of the computer consists of a linear scaling multiplier which magnifies the process output deviation to an amplitude which can be handled accurately in the nonlinear computer circuits that follow; a correlating multiplier-integrator (the integrator is capable of being reset to zero at the end of each complete identification cycle); several sample and hold circuits for storage of required information; and a multiplier and a divider to generate the estimate of the process gain and the correct amplitude of the sinusoid which is used to perturb the process.

All of the dual-mode equipment was constructed with relays. The mode of the relay determines what mode the computer stage is in (for example reset or integrate for the integrator). It is then possible to change the mode of any stage momentarily by pulsing the corresponding relay coil with a pulse of adequate magnitude and duration. The logic circuits produce such pulses at the proper switching times.

Operation. The operation of the identification computer corresponds exactly to the identification algorithm given in the theoretical section. If we assume that the system has been in operation for some time, the actual sequence of events is as follows. The correlating integrator is reset to zero momentarily, just as the identification period is completed and the process output assumes its average value. Evaluation of the correlating integral given by Equation (22) proceeds in real time for one full cycle of the output deviation. During this time sample/hold 1 stores or holds the value of the integral obtained during the previous cycle Z_{n-1}, and sample/hold 2 stores the value of process gain used in the previous interval. These quantities are used to estimate the present value of process gain k_n as given by Equation (26). This value of k_n maintains the value of the input sinusoidal perturbation at A_n as given in Equation (27).

Just at the end of this cycle, a new value of Z is obtained Z_n, and the logic circuits compute instantaneously how good

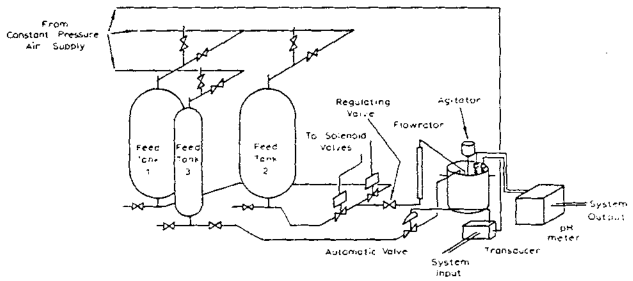

Fig. 6. Schematic diagram of the identification flow system.

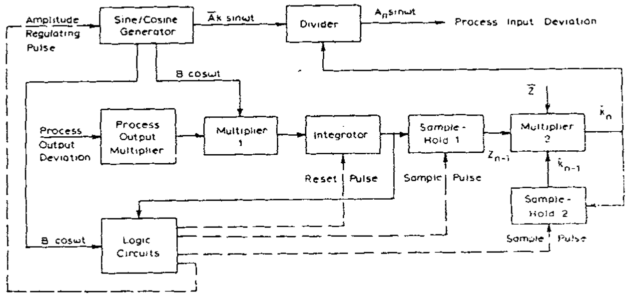

Fig. 7. Block diagram of identification computer.

an estimate $\hat{k}_n$ was of k_n by determining how close Z_n is to $\tilde{Z}$. If $(1 - p)\tilde{Z} \leq Z_n \leq (1 + q)\tilde{Z}$, where p and q are positive fractions used to set the allowable variation* in Z_n, the logic circuits conclude that the old value of $\hat{k}_n$ is still a good estimate and simply reset the integrator to zero. If Z_n is outside these limits, sample/hold 2 is pulsed which brings in $\hat{k}_n$ as $\hat{k}_{n-1}$, sample/hold 1 is pulsed which brings in the new value of Z_n, and then the integrator is reset to zero. The process proceeds in this manner. Furthermore, the logic circuits maintain the amplitude of both outputs of the sine/cosine generator constant by setting them to their proper values once per cycle. Thus, all operations are self-sustaining and no external influence is required.

Complete circuit diagrams and detailed descriptions of all analog circuitry may be found in reference *4*.

EXPERIMENTAL

A number of step tests were performed to characterize flow system dynamics over a range of total flow rates. At a total flow rate $(G + \bar{g})$ of 1 liter/min. a basic system apparent time constant of 182 sec. was obtained. A frequency response test indicated an apparent dead time of 1.5 sec. The theoretical dynamics would predict a basic time constant of 180 sec. The approximate transfer function describing the system is therefore

$$\frac{pH^*(s)}{g^*} = \frac{k'e^{-1.5s}}{182\,s + 1} \tag{29}$$

* Experimental evidence indicates that a certain amount of variation in Z_n exists even when the system is stationary (gain is constant). This approximate 5% variation in Z_n results from low frequency noise. The limits of $Z_n(p,q)$ are used to prevent overcompensation of the input deviation amplitude when the system gain is not changing.

where $k' = SK_p$, and K_p is the stationary process gain as derived earlier. The flow rate g^* was a linear function of the actual computer input to the process (v.) related by the valve and transducer coefficients. Both these devices were linear.

Conversion of pH to a computer input (v.) was accomplished by amplifying the pH measuring device output (mv.). The resulting quantity, related to pH by the expression

$$V = 8.52\,pH + V_o \tag{30}$$

was the identification process output. Hence, values of S will be reported in the units of volts per milliliter of base per liter of total solution. The conversion to pH units may be accomplished by means of Equation (30).

Identification Tests with a Simulated Flow System

Initial tests of the identifier were made with an analog simulation of Equation (29) with constant gain to test the ability of the identifier to estimate the gain of a stationary process. The results of these tests showed that the identifier could measure the system gain accurately.

Identification Tests with the Actual Flow System

A series of tests was made to characterize the identifier when operated in conjunction with a real system. Phosphoric acid solutions were used as the feedstock with 0.48M KOH as the control reagent. Both stationary and nonstationary operations of the flow system were used. In the stationary tests a feedstock of constant buffer species concentration x' was used. To test the identifier with a nonstationary system, two feedstocks with different values of x' were used. The input to the system was changed suddenly from one solution to the other by means of the solenoid valves.

A range of x' (total phosphoric species concentration) of 0.005 to 0.02 g./moles/liter was used in these tests. The system operating point was in the vicinity of pH 7. The range

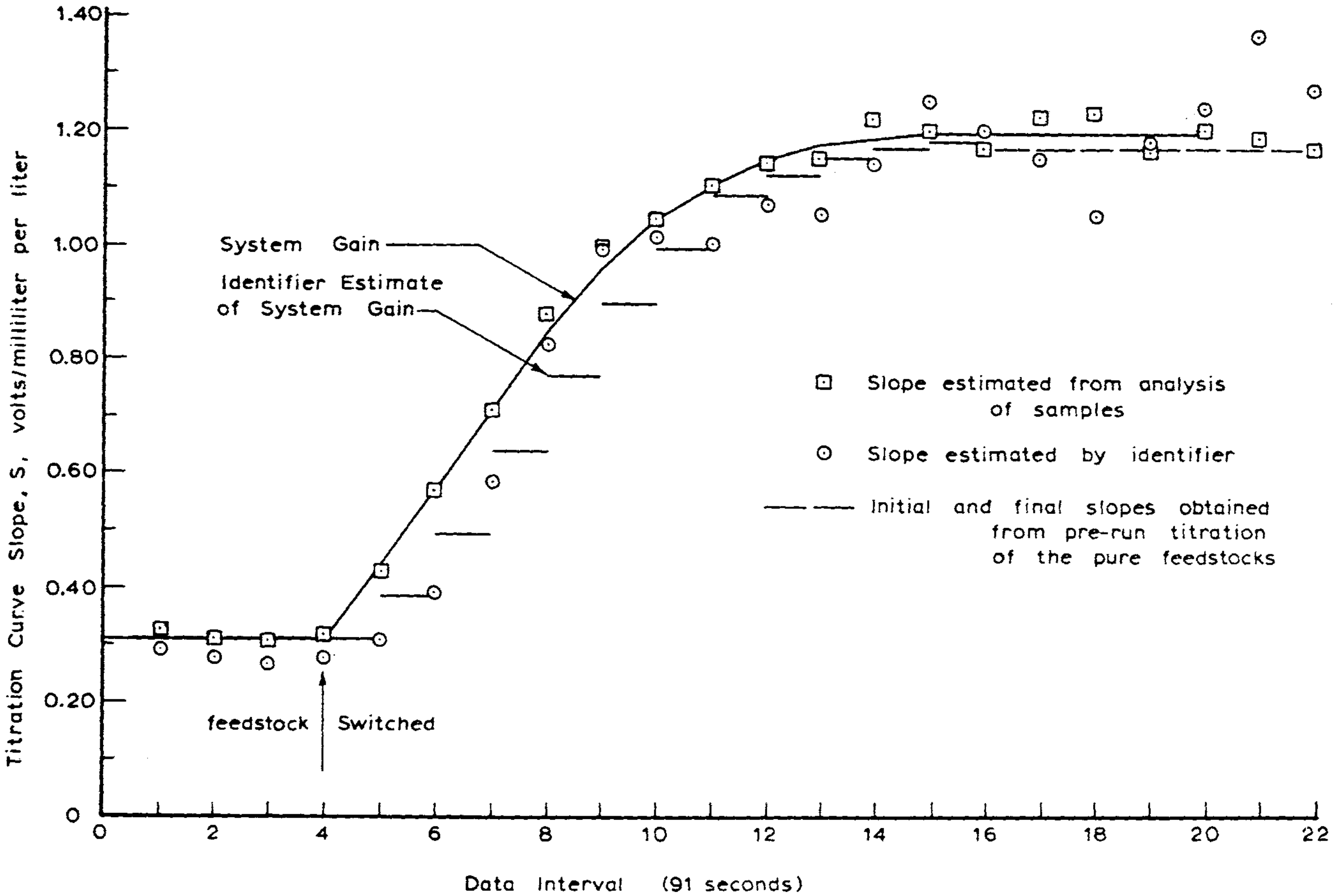

Fig. 8. Identification of flow system gain during nonstationary operation. Solid lines indicate analog computer simulation of test.

of variation of S under these conditions was approximately 4:1. In the runs involving two different feedstocks, the operating point was determined as the intersection point of the two titration curves so as to eliminate changes in pH after the solution switch was made.

During one of the nonstationary runs (run 17) the contents of the stirred tank were sampled and later analyzed to estimate the actual value of S as a function of time. This estimate of S from a sample was obtained by adding a measured amount of base to the sample and noting the change in pH.

The results of the stationary tests again showed the identifier to be accurate.

Figure 8 is presented to show graphically the relationship between system gain and the identifier estimate of gain for the nonstationary portion of run 17. The identifier response appears to lag the actual system gain by some constant value, although it is difficult to estimate the delay exactly because of some scatter in the data.

A similar identifier response to changing system gain was seen in additional tests of this type. The scatter seen in the identifier estimates is the result of small fluctuations in the system output average value. In the derivation of the identification theory, $\bar{y}$, the quantity corresponding to $\overline{pH}$, was assumed constant. Because of the low level of the output deviation amplitude A (0.1 pH, which was chosen this low to provide a severe test of the method), small, low frequency fluctuations in $\overline{pH}$ of 0.02 pH unit gave rise to some scatter in the identifier estimates.

To estimate better the identifier dynamics in response to gain variations of this nature, the analog computer was used to simulate these transient tests. The results of the simulation of run 17 are also given in Figure 8. From these results the identification delay is seen to be very close to $T_k/2$. This is the same value as was obtained by analytical methods when a linear process gain variation was assumed.

CONCLUSIONS

1. A class of multivariable control problems has been considered. By means of an example, the reduction of such a problem to a single input/single output system with a time varying parameter, the process gain, has been described.
2. A practical identifier has been built to estimate the time varying process gain continuously. The identifier offers the following advantages: (1) It operates automatically, that is, is completely self-sustaining. (2) The system output perturbation level is maintained constant by varying the input perturbation amplitude so as to maintain accuracy in gain estimation. (3) Because an integral method is used, high frequency fluctuations in the process output are smoothed out. (4) The response to a changing system gain is good, particularly when compared to stochastic identification procedures. The identifier obtains a new estimate of system gain every $T_I/2$ time units where T_I is the dominant time constant of the identification tank.
3. The disadvantages of the method are: (1) The identifier furnishes an estimate of the process gain only. (2) A perturbation of the identification process is necessary. (3) As will be pointed out in the companion paper, separate identification and control flow systems are required to eliminate the effect of loads on the system.

NOTATION

A = amplitude of the sinusoid used as the identification system input perturbation
B = amplitude of the correlating cosine term in Equation (21)
C = amplitude of the correlating sine term in Equation (21)
C_B = concentration of base, equivalents/liter
c_B = normalized concentration of strong base, volume base/volume total solution
c_{BL} = normalized concentration of strong base in the load stream
c_{BM} = normalized concentration of strong base in manipulated stream
D = approximate amplitude of $z^*(t)$, defined by Equation (24)
G = stream volumetric flow rate, liter/min.
$G_c(s)$ = controller transfer function
g = stream volumetric flow rate, liter/min.
K_c = controller gain
K_L = stationary portion of the process gain relating the output to the load variable
K_p = stationary portion of the process gain relating the output to the manipulated variable
k = process gain
n = value associated with the n^{th} sampling interval
P = linear rate of change of process gain
p = dimensionless constant expressing the linear rate of change of identification system gain
p = fractional quantity fixing the lowest value which Z_n can take on and not cause the identifier to recompute a new value of k_n
pH_s = steady state value of pH
pH_c = pH of the material in the measurement cell
pH_m = measured value of pH
Q = initial value of process gain
S = slope of the pH vs. c_b curve at the operating point (pH units/(ml.)(liter) or v./(ml.)(liter)
T = system basic time constant, sec.
T_D = delay time
T_m = time constant of a pH measurement cell
T_k = length of an identification interval
t = time
V = tank volume, liters
V = computer voltage related to pH, defined by Equation (30)
V_{pipe} = volume of pipe between control tank and measurement, cell/liters
X = vector consisting of the concentrations of all buffer species in an aqueous solution, g.-moles/liter
X' = vector defined by Equation (2)
x = input variable of a general identification system
x' = X' when only a single weak acid is present
x_B = strong base concentration, g.-mole/liter
y = output variable of a general identification system
Z = value of the definite integral defined by Equation (22)
Z' = value of the integral defined by Equation (21)
z = intermediate variable of a general identification system
ω = sinusoidal frequency, radians/sec.

Superscripts

$*$ = deviation variable
$-$ = steady state value
$\sim$ = value associated with the desired value of Z
$\wedge$ = estimated value

LITERATURE CITED

1. Marcikic, Sava, M.S. thesis, Purdue Univ., Lafayette, Ind. (1962).
2. Angus, R. H., and Leon Lapidus, *A.I.Ch.E. J.*, 9, 810 (1963).
3. Box, G. E. P., and Jayarajan Chanmugam, *Ind. Eng. Chem. Fundamentals*, 1, 2 (1962).
4. Mellichamp, D. A., Ph.D. thesis, Purdue Univ., Lafayette, Ind. (1964).

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